

04A: Laboratory measurement of Deformation at High Temperatures

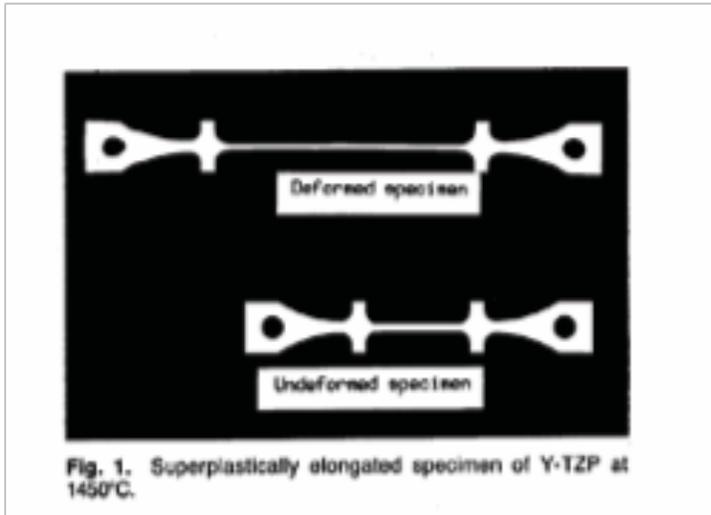
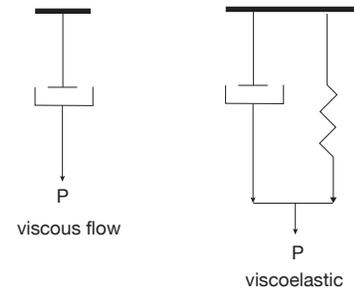


Figure 1

A sample of zirconium oxide, nominally a brittle ceramic, deformed to large elongation at high temperatures. This phenomenon is called superplasticity. More like viscous flow.

Three Aspects

- A. Laboratory measurements of deformation at high temperature
- B. Atomistic Mechanism and the role of solid-state diffusion
- C. Model that coalesces the influence of stress, temperature, and the microstructure.



A: Laboratory Measurements of Deformation at High Temperature

The Tensile Test I: True Stress and True Strain (and Strain Rate)

In elastic deformation the strains are small ($<1\%$). In plastic yielding also the tensile strains are rarely larger than 10% . However, in superplastic deformation, as seen in Fig. 1, the strains can be very large, often greater than 100% , which requires the strains and stresses to be expressed in terms of true strain in true stress. These expressions are derived from two fundamental equations. The true strain

$$\varepsilon = \ln \frac{L}{L_o}, \quad \varepsilon = \ln \frac{L_o + \Delta L}{L_o} = \ln \left(1 + \frac{\Delta L}{L_o} \right) = \ln(1 + \varepsilon_{eng})$$

If $\varepsilon_{eng} < 0.1$, then since $\ln(1+x) = x + x^2 \text{ etc.}$ $\varepsilon = \ln(1 + \varepsilon_{eng}) \approx \varepsilon_{eng}$

Therefore, must consider $\varepsilon = \ln \frac{L}{L_o}$ for superplastic deformation. $\left(\frac{L}{L_o} = e^\varepsilon \right)$ (1A)

where L is the elongated length and L_o is the original gage length. The stress depends on the change in the cross section with strain (from A_o to A). They are related by the constant volume condition

$$A_o L_o = AL \quad (1B)$$

Therefore, the tensile strain may also be written in terms of the change in the cross section

$$\varepsilon = \ln \frac{A_o}{A} \quad (1C)$$

Let us now consider a tensile test carried out in an Instron where strain is applied to the specimen by the displacement of the crosshead and the load is measured with an in-line load cell. The high temperature-strain rate experiment is carried out in the following way

- i. The sample is supported by "grips" and placed within the furnace
- ii. The furnace is brought up to a constant temperature
- iii. The crosshead is moved at a prescribed rate, while the load is measured with an in-line load cell.

The question arises how the velocity of the crosshead needs to increase with time to keep pace with the greater length of the sample, in order to maintain a constant effective strain rate. Differentiating Eq. (1A) with respect to time

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = \frac{1}{L} \frac{dL}{dt} = \frac{\dot{L}}{L} \quad (1D)$$

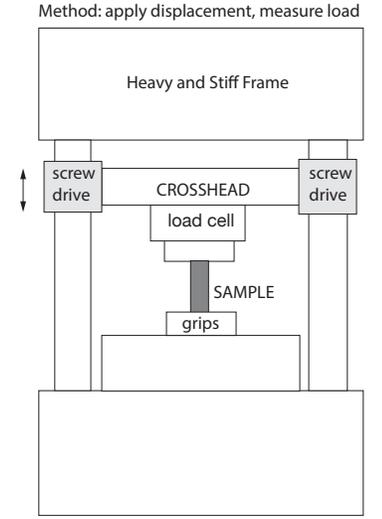
Therefore to maintain a constant strain rate \dot{L} must increase with time according to

$$\dot{L} = L(t)\dot{\varepsilon} \quad (1E)$$

Now, $\dot{L} = L(t)\dot{\varepsilon}$ the initial strain when the specimen length is L_o and $\varepsilon = 0$ is a limiting case of Eq. (1E)

$$\dot{L}_o = L_o(t)\dot{\varepsilon} \quad (1F)$$

Dividing both sides of (1E) by (1F) we have that



$$\frac{\dot{L}}{\dot{L}_o} = \frac{L(t)}{L_o} \quad (1G)$$

substituting from (1A), we get the final result

$$\frac{\dot{L}}{\dot{L}_o} = e^\varepsilon \quad (1F)$$

Now, the true stress is no longer equal to (P / A_o) since the cross-sectional area is decreasing with strain. The true stress is given by

$$\sigma = \frac{P}{A} = \frac{P}{A_o} \frac{A_o}{A}$$

Substituting from Eq. (1C) we get

$$\sigma = \sigma_{eng} e^\varepsilon \quad (1G)$$

where σ_{eng} is the engineering stress.

Time until the specimen is elongated by 100%, that is, to twice its original length at a strain rate of 10^{-4} sec^{-1} . 100% engineering strain means a true strain of $\ln 2 = 0.7$. Therefore the time of the experiment = $0.7/0.0001 \text{ s} = 7,000 \text{ s}$ which is about 120 min or two hours.

The Tensile Test II: The Strain Rate Equation

Phenomenological Relationship between stress, strain rate, temperature, and microstructure. This relationship is generally given by:

Figure 2

$$\dot{\varepsilon} = A \frac{\sigma^n}{d^p} e^{-\frac{Q}{RT}} \quad (1)$$

σ is the tensile stress

d is the grain size (microstructure)

T is the temperature in Kelvin

Q Activation Energy

Separation of variables means that the influence of each variable on the strain rate can be studied experimentally, independently of the other variables.

For example, the influence of stress can be measured by conducting experiments at different stress levels, but at the same temperature.

Similarly, the influence of temperature can be examined by experiments carried out at different temperatures, but at the same stress.

The Stress Dependence (represented by "n" called the power law)

Let us consider first the measurement of the stress dependence. Since this dependence may be non-linear, that is $n > 1$, a logarithmic plot can be used to determine its value as in Eq. (1). Taking the natural logarithm of both sides (since the expression contains an exponential term) we obtain

$$\ln \dot{\epsilon} = \ln A + n \ln \sigma - \frac{Q}{RT} - p \ln d \quad (2)$$

It is better to work with logarithm to the base 10, then

$$\log_{10} \dot{\epsilon} = \log_{10} A + n \log_{10} \sigma - \frac{Q}{2.3RT} - p \log_{10} d \quad (3)$$

The factor of 2.3 in the denominator in $Q/(2.3RT)$ arises because

$$\ln_e(x) = \log_{10}(x) \ln_e(10)$$

Therefore

$$\log_{10}(x) = \frac{\ln_e(x)}{2.3}$$

Returning to Eq. (2), if the temperature and the grain size remain constant, and since A is also a constant, we have that

$$\log_{10} \dot{\epsilon} = \text{Constant} + n \log_{10} \sigma \quad (5)$$

The data for the experiment shown in Fig. 2 above, in a log-log plot is shown on the right. The "power law", n, in Eq. (5), tells the orders of magnitude increase in the strain rate for one order of magnitude increase in the stress. The triangle in purple color on the right shows what to expect if $n=2$. The data show that in-fact $1 < n < 2$. If one measures it exactly one finds that the strain rate increases by about 1.5 orders of magnitude for one order of magnitude increase in the stress. The numerical value for a factor of 1.5 orders of magnitude will be $= 10^{1.5}$, that is ~ 30 , remember that $10^2 = 100$.

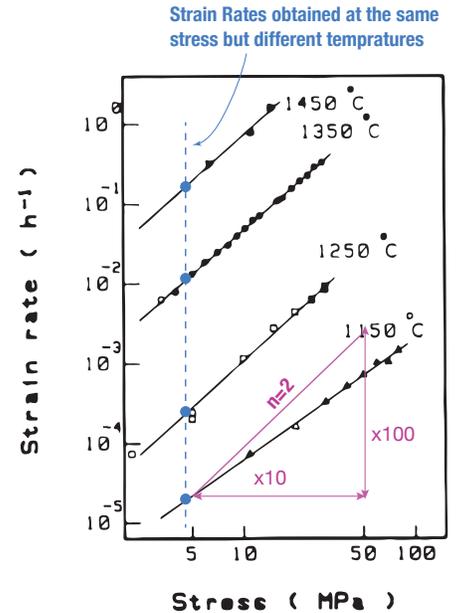


Fig. 3. Steady-state creep rate vs applied stress for Y-TZP